Hitting in: a probabilistic approach

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I shall denote your probability of hitting a target k balls wide at a distance d yards by $P(d,k)$ and I shall assume that you have taken a large number n of 13-yard shots at a single ball and have made r roquets, thereby estimating $P(13,1)$ as r/n.

That's straightforward enough with the possible exception of what I mean by "a target k balls wide at a distance d yards". You can make an angular error of $\pm 1/d$ ball widths and still hit a single ball; think of the centre of striker's ball and how far off-line it can be and still make a roquet. For two touching balls you can make an error of $\pm 1½/d$ ball widths so I denote the probability of hitting one (or both) of them by $P(d,1½)$. For a 'perfect double', that is two balls exactly a ball apart, you can make an error of $\pm 2/d$ ball widths so that is $P(d,2)$; it really is a double target.

From $P(13,1)$ it is possible to calculate the probabilities of hitting these other shots. $P(13,1½)$ is interesting because that is what you need when shooting from the end of B-baulk at two touching balls on the west boundary in corner II. That is easier to hit than two corner balls on the north boundary but your opponent may chose to leave you the larger target as hitting it probably won't yield a cannon whereas the alternative almost always will. I will also look at a perfect double at 13 yards and at different lengths of shot at double targets with gaps wide enough to let a ball through, though I shall restrict the other lengths to 8 and 18 yards. It is reasonably easy to generalise from these three distances. For completeness I shall also include the probability of hitting the peg, for which $k = 41/58$.

First I have to make some assumptions; I won't try to justify them except to say that while they are almost certainly wrong they don't seem too stupid. Firstly I assume that an individual's angular error θ doesn't depend on the length of shot, though I'm pretty sure mine increases the harder I hit; and secondly I assume it has a logistic distribution. This is similar to the well-known Gaussian distribution but much more tractable mathematically.

Naturally there could be discussion about the assumptions: the error will vary from lawn to lawn, it might be higher and not symmetric near some boundaries, you might not have the same probabilities in a match as in practice, nerves might affect you at some lengths more than others, and so on. However, the probabilities I shall derive can inform your choice of shot, both what you might take on and what you might leave your opponent. I shall present results where $P(13,1)$ varies from 0.3 to 0.7.

It will be convenient to denote $\{1 - P(d,k)\} / \{1 + P(d,k)\}$ by $\varphi(d,k)$ which formula can be rearranged to show that $P(d,k) = \{1 - \varphi(d,k)\} / \{1 + \varphi(d,k)\}$.

The logistic distribution says that the probability that your angular error is less than x, $\Pr(\theta < x) = 1 / \{1 + \exp(-x/a)\}$ where a is a constant related to your variability. From that it is easy to show that $\Pr(|\theta| < x) = \{1 - \exp(-x/a)\} / \{1 + \exp(-x/a)\}$. If we use b to denote the diameter of a ball, then to hit a single ball at 13 yards you need $|\theta| < b/13$ and that means that $\exp(-b/13a) = \varphi(13,1)$. Since $\varphi(d,k) = \varphi(13,1)$ **raised to the power** $13k/d$, we can express the probability of hitting a target of any width at any distance in terms of $P(13,1)$.

Suppose now there is a gap of greater than the width of a ball between the two balls you are aiming at and you shoot at the middle of the gap. Call the probability of hitting $Q(d,g)$ where g is the gap in ball diameters. For $g > 1$, $Q(d,g) = P(d,2+½(g-1)) - P(d, ½(g-1))$, in other words the probability of getting the ball within the target area but not going through the gap. That too is a function only of $P(13,1)$.

Now we must calculate the probability of hitting if you aim at one of the balls. Imagine you are aiming at the right hand ball. If there was yet another ball to its right the same distance away as the one to the left then we would have symmetry again. Take away the middle ball. You are now aiming at the centre of the gap and that is the situation we have just dealt with. We can therefore write the new probability **as**

$$Q'(d,g) = P(d,1) + \tfrac{1}{2}Q(d,2g+1) = P(d,1) + \tfrac{1}{2}\{P(d,2+g) - P(d,g)\},$$

which is therefore once again a function only **of P(13,1)**.

Some key results have been collected in the table; it shows percentages rather than probabilities.

8 yards	1 ball	P(8,1)	46	53	60	66	71	76	81	85	88
	1½ balls (touching)	P(8,1½)	64	71	77	83	87	91	93	96	97
	2 balls (perfect double)	P(8,2)	76	83	88	92	95	96	98	99	99
	gap of 2 balls: centre	Q(8,2)	60	61	61	59	56	52	48	44	39
	gap of 2 balls: ball	Q′(8,2)	56	61	65	70	74	78	82	86	89
	peg	P(8,$^{41}/_{58}$)	34	40	45	51	56	61	66	71	76
13 yards	**1 ball**	P(13,1)	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**
	1½ balls (touching)	P(13,1½)	43	50	56	62	68	73	78	82	86
	2 balls (perfect double)	P(13,2)	55	62	69	75	80	84	88	91	94
	gap of 2 balls: centre	Q(13,2)	50	54	58	60	61	61	61	59	57
	gap of 2 balls: ball	Q′(13,2)	45	49	52	56	59	62	65	69	73
	peg	P(13,$^{41}/_{58}$)	22	25	29	33	37	41	45	50	55
18 yards	1 ball	P(18,1)	22	26	30	34	38	42	46	51	56
	1½ balls (touching)	P(18,1½)	32	38	43	48	53	58	64	69	74
	2 balls (perfect double)	P(18,2)	42	48	55	60	66	71	76	81	85
	gap of 2 balls: centre	Q(18,2)	40	45	49	53	56	59	60	61	61
	gap of 2 balls: ball	Q′(18,2)	37	41	44	48	51	54	56	59	62
	peg	P(18,$^{41}/_{58}$)	16	18	21	24	27	31	34	38	42

What conclusions should we come to? If your **P(13,1)** is less than a half you would be advised to shoot at the middle of 13 yard (or longer) doubles with a gap of two balls, unless you are so keen to hit a particular ball that you overrule the probabilities; if you are better than that, choose a ball at 13 yards, but the gap at 18 yards. At 8 yards choose a ball unless you hit fewer than one in three 13-yard shots.

Since 41/58 is very close to 13/18 the probability of hitting the peg from 13 yards is almost exactly the same as hitting a ball at 18 yards – well-known but worth remembering.

One final technicality. Sometimes your optimal aiming point is neither the middle of the gap nor a ball. For example, if your **P(13,1)** is 0.55, shooting at the middle of the gap from 13 yards offers you a probability of 0.61 and aiming at one of the balls gives 0.62, but shooting at the inner edge of one of the balls increases that to 0.64. Not a huge gain, only relevant over a very small range of accuracies, and possibly dependent on the distribution we have chosen, but of some slight theoretical interest.

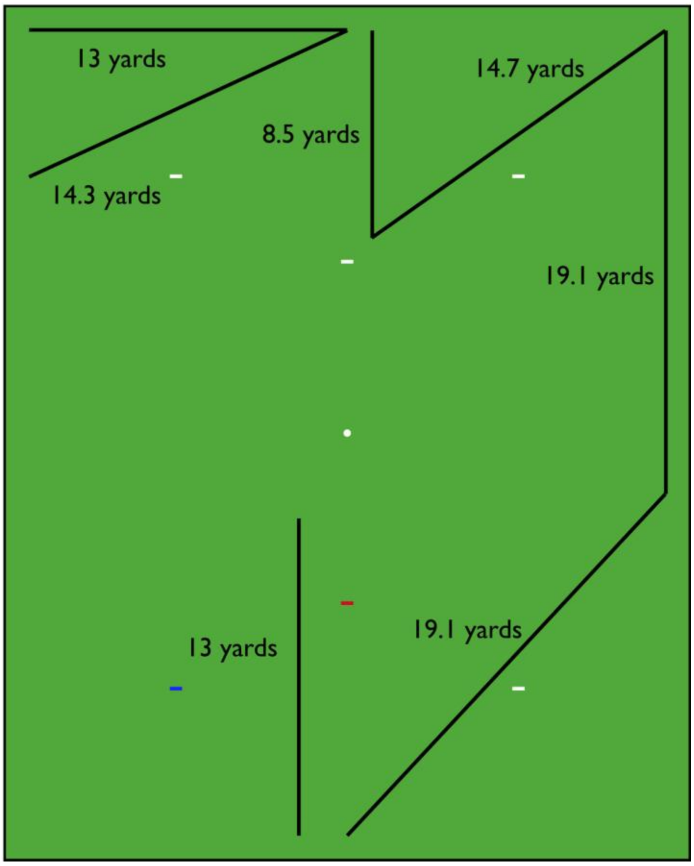
For values other than those shown in the table there is an interactive calculator written by Campbell Morrison on the SCA website at http://www.scottishcroquet.org.uk/roquet/ .

If you would like to investigate such things you are welcome to make use of the JavaScript implementation of a function to calculate the probability of a roquet which is available at http://www.scottishcroquet.org.uk/roquet/roquet.js . I have found it more elegant when programming the general case of the double target to think in terms of one ball being displaced from the other a certain number of ball diameters, rather than measuring the gap between them.

I shall be glad to receive any comments from coaches or players (or statisticians).

Your **P(13,1)** is related to $d_{½}$, the distance at which you hit half of the time. Unless you are a very accurate shot a good approximation to the relationship is $d_{½}$ = **26P(13,1)**.

Finally, here is a reminder of the lengths of some frequently encountered shots.



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